

International Journal of Modern Physics A  
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## ULTRA-HIGH ENERGY NEUTRINO DISPERSION IN PLASMA AND RADIATIVE TRANSITION $\nu_L \rightarrow \nu_R + \gamma$

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Received Day Month Year

Revised Day Month Year

Qualitative analysis of additional energy of neutrino and antineutrino in plasma is performed. A general expression for the neutrino self-energy operator is obtained in the case of ultra-high energies when the local limit of the weak interaction is not valid. The neutrino and antineutrino additional energy in plasma is calculated using the dependence of the  $W$  and  $Z$ -boson propagators on the momentum transferred. The kinematical region for the neutrino radiative transition (the so-called “neutrino spin light”) is established for some important astrophysical cases. For high energy neutrino and antineutrino, dominating transition channels in plasma,  $\nu_e + e^+ \rightarrow W^+$ ,  $\bar{\nu}_e + e^- \rightarrow W^-$  and  $\bar{\nu}_\ell + \nu_\ell \rightarrow Z$ , are indicated.

*Keywords:* neutrino; self-energy operator; spin light; external active medium; supernova

PACS numbers: 13.15.+g, 95.30.Cq

### 1. Introduction

The neutrino physics development during the last decades, and especially solving the Solar neutrino puzzle in the unique experiment on the heavy-water detector at the Sudbery Neutrino Observatory together with monitoring the Galaxy by the net of neutrino detectors aimed to registrate a neutrino signal from the expected galactic supernova explosion, brings to the fore the neutrino physics in an active external medium. The study of the external medium influence on the neutrino dispersive properties is based on the analysis of the neutrino self-energy operator.

The neutrino self-energy operator  $\Sigma(p)$  can be defined in terms of the invariant amplitude for the transition  $\nu \rightarrow \nu$ , that is the neutrino coherent forward scattering<sup>1</sup>, by the relation:

$$\mathcal{M}(\nu \rightarrow \nu) = -[\bar{\nu}(p)\Sigma(p)\nu(p)] = -\text{Tr}[\Sigma(p)\rho(p)], \quad (1)$$

where  $p^\alpha = (E, \mathbf{p})$  is the neutrino four-momentum,  $\rho(p) = \nu(p)\bar{\nu}(p)$  is the neutrino density matrix. Effect of the external active medium on neutrino properties specifies

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an appearance of the additional neutrino energy that can be defined via the self-energy operator  $\Sigma(p)$  as follows:

$$\Delta E = \frac{1}{2E} \text{Tr}[\Sigma(p)\rho(p)]. \quad (2)$$

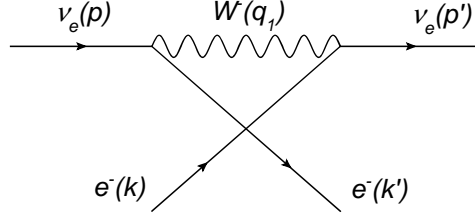
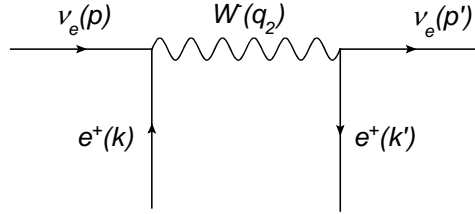
It should be mentioned that the medium influence on neutrino properties is due primarily to the additional energy acquired only by the left-handed neutrinos. The discovery of the neutrino oscillations and hence of non-zero neutrino masses points to the necessity of existence of the right-handed neutrinos, which are sterile to the weak interactions and therefore are not acquiring additional energy in medium. If a neutrino carries a magnetic moment, there exists a possibility for interaction with photons leading to the neutrino spin flip. In this case the left-handed neutrino additional energy appearance makes possible the neutrino radiative conversion:

$$\nu_L \rightarrow \nu_R + \gamma. \quad (3)$$

This situation called the “spin light of neutrino” ( $SL\nu$ ), was first proposed and investigated in detail in an extended series of papers (see Ref. 2 and the papers cited therein). However, in the analysis of this effect the authors missed the plasma influence on the photon dispersion. As it was shown in Refs. 3 and 4, taking account of this influence makes the neutrino spin light process kinematically forbidden in almost all real astrophysical situations. In the latest publications (see e.g. Ref. 5), a consideration of the  $SL\nu$  process reduced to the limit of ultra-high neutrino energies. Actually, in this case the dispersion properties of a photon can be neglected. But the using of the weak interaction local limit would not be justified then.

There exists another physical possibility where the expression for the neutrino additional energy in plasma obtained in the local limit of the weak interaction is insufficient. It occurs in the case of nearly charged-symmetric plasma, e.g. in the conditions of the Early Universe. In this case the local contribution to the neutrino additional energy vanishes, and a part of the neutrino additional energy caused by non-locality of the weak interaction becomes essential. This contribution to the neutrino additional energy was investigated in Ref. 6 (see also Refs. 7 and 8).

In the listed papers <sup>6,7,8,4</sup> the accounting of the non-local contribution to the neutrino additional energy was made by the retention of the next term in the expansion of the  $W$ - and  $Z$ -boson propagators in the inverse powers of their masses. However in the limit of the ultra-high energies this kind of expansion should be banned and therefore it is necessary to use the exact expressions for the  $W$ - and  $Z$ -boson propagators. Analysis of the neutrino additional energy in a plasma in the limit of ultra-high energies, with taking account of the nonlocality of the weak interaction was made in a series of papers, Refs. 9, 10, 11, with respect to the neutrino oscillations. In the present paper we consider the neutrino self-energy operator in medium with taking into account the dependence of the  $W$  and  $Z$ -boson propagators on the momentum transferred, and we analyse its effects on the neutrino radiative conversion (3).

Fig. 1. The Feynman diagram for the neutrino-electron scattering through  $W$ -boson.Fig. 2. The Feynman diagram for the neutrino-positron scattering through  $W$ -boson.

## 2. Neutrino Self-Energy Operator in Medium

Let us consider first the electron neutrino scattering on the electron-positron component of plasma.

The Lagrangian of the interaction has the form:

$$L = \frac{g}{2\sqrt{2}} [(\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e)W^\alpha + \text{h.c.}], \quad (4)$$

where  $\gamma_5$  is used in notations of Ref. 12, and leads to the invariant amplitude of the process:

$$\begin{aligned} M_{\nu_e e^- \rightarrow \nu_e e^-} &= -\frac{G_F}{\sqrt{2}} [\bar{e}(k')\gamma_\alpha(1 - \gamma_5)e(k)] \\ &\times [\bar{\nu}_e(p')\gamma^\alpha(1 - \gamma_5)\nu_e(p)] \frac{1}{1 - q_1^2/m_W^2}, \end{aligned} \quad (5)$$

where we use the notation  $q_1 = k - p'$  for the  $W^-$ -boson momentum (see Fig. 1). Here, the Fiertz transformation is performed, and the small term in the  $W$ -boson propagator of the order of  $(m_e/m_W)^2$  is neglected.

The amplitude of the neutrino-positron scattering process can be written in the similar form (see Fig. 2):

$$\begin{aligned} M_{\nu_e e^+ \rightarrow \nu_e e^+} &= \frac{G_F}{\sqrt{2}} [\bar{e}(-k)\gamma_\alpha(1 - \gamma_5)e(-k')] \\ &\times [\bar{\nu}_e(p')\gamma^\alpha(1 - \gamma_5)\nu_e(p)] \frac{1}{1 - q_2^2/m_W^2}, \end{aligned} \quad (6)$$

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where  $W^-$ -boson momentum is  $q_2 = -p - k$ . Note that in contrast to the  $u$ -channel process, described by the diagram in Fig. 1, the process in Fig. 2 is of the  $s$ -channel type. It means that in this process, a resonance behavior of the  $W$ -boson propagator manifests itself. Taking account of this type of resonance is made by introducing a complex mass of  $W$ -boson,  $m_W^* = m_W - \frac{1}{2}i\Gamma_W$ , where  $\Gamma_W$  is the total decay width of  $W$ -boson,  $\Gamma_W \simeq 2.1$  GeV.

Because of the  $t$ -channel behavior of the neutrino-electron and neutrino-positron scattering diagrams for neutrinos of all flavors through  $Z$ -boson, and keeping in mind that the forward scattering is considered, i.e. the scattering with zero-momentum transfer, one concludes that the contribution to the energy from these subprocesses is described by the local limit of the weak interaction.

The total contribution to the neutrino self-energy operator for  $\ell$ -flavor neutrino from the neutrino scattering processes on plasma electrons and positrons can be represented in the form:

$$\Sigma_{(e^-e^+)}^{\nu_\ell}(p) = \sqrt{2}G_F [C_V(u\gamma)\gamma_L(N_e - \bar{N}_e) + \delta_{\ell e}\gamma^\alpha\gamma_L(j_\alpha^- - j_\alpha^+)], \quad (7)$$

where  $\gamma_L = (1 - \gamma_5)/2$ ,  $N_e, \bar{N}_e = 2(2\pi)^{-3} \int d^3k (\exp((\varepsilon \mp \mu)/T) + 1)^{-1}$  are the electron and positron densities respectively, and we use the notation

$$j_\alpha^\mp = 2 \int \frac{d^3k}{(2\pi)^3} \frac{k_\alpha}{\varepsilon} \left( e^{\frac{\varepsilon \mp \mu}{T}} + 1 \right)^{-1} \left( 1 \pm \frac{2(kp)}{m_W^2} \right)^{-1}. \quad (8)$$

The constant  $C_V$  in Eq. (7) comes from the electron  $Z$ -current,  $C_V = -1/2 + 2 \sin^2 \theta_W$ , where  $\theta_W$  is the Weinberg angle.

In accordance with Eq. (2), the neutrino  $\nu_\ell$  additional energy in the electron and positron medium takes the form:

$$\Delta E_{(e^-e^+)}^{\nu_\ell} = \sqrt{2}G_F [C_V(N_e - \bar{N}_e) + \delta_{\ell e}(F_1(\mu_e, m_W) - F_2(-\mu_e, m_W))], \quad (9)$$

where we introduce the functions

$$F_{1,2}(\mu, m) = \frac{2}{(2\pi)^3 E} \int \frac{d^3k}{\varepsilon} \left( e^{\frac{\varepsilon \mp \mu}{T}} + 1 \right)^{-1} \frac{(pk)}{\left( 1 \pm \frac{2(pk)}{m^2} \right)}. \quad (10)$$

In order to obtain the antineutrino additional energy in the same medium, one has to make the replacement  $\mu_e \rightarrow -\mu_e$  in the right-hand side of Eq. (9). In the first term with the difference of the electron and positron densities it simply means a change of sign.

In the analysis of the neutrino dispersion in active astrophysical medium in a general case, the presence of the other plasma components, protons and neutrons, must be considered. In a dense plasma of the supernova core the donation from thermal neutrinos that can be considered to be approximately in equilibrium, can also be significant. The two of the four Feynman diagrams for the neutrino-neutrino interaction contain a contribution from the non-locality of weak interaction.

A complete formula for the  $\nu_\ell$  neutrino and  $\bar{\nu}_\ell$  antineutrino additional energy can be written in the following way:

$$\begin{aligned} \Delta E^{\nu_\ell, \bar{\nu}_\ell} = & \sqrt{2} G_F \left\{ \mp \frac{1}{2} (N_n - \bar{N}_n) \pm (N_{\nu_e} - \bar{N}_{\nu_e}) \right. \\ & \pm (N_{\nu_\mu} - \bar{N}_{\nu_\mu}) \pm (N_{\nu_\tau} - \bar{N}_{\nu_\tau}) \\ & + \delta_{\ell e} [F_1(\pm \mu_e, m_W) - F_2(\mp \mu_e, m_W)] \\ & \left. + \frac{1}{2} [F_1(\pm \mu_{\nu_\ell}, m_Z) - F_2(\mp \mu_{\nu_\ell}, m_Z)] \right\}. \end{aligned} \quad (11)$$

In this expression,  $N_n, N_{\nu_\ell}$  are the neutron and neutrino densities and  $\bar{N}_n, \bar{N}_{\nu_\ell}$  are the densities of the corresponding antiparticles. Electron and proton densities are cancelled in Eq. (11) because of plasma electroneutrality. Note that in both functions  $F_2$  there exists the mentioned above resonance behavior, which can be accounted by the introduction of complex masses of  $W$ - and  $Z$ -bosons,  $m_{W,Z}^* = m_{W,Z} - \frac{1}{2} i \Gamma_{W,Z}$ , where the total decay width of the  $Z$ -boson is  $\Gamma_Z \simeq 2.5$  GeV.

Tending formally  $m_W$  and  $m_Z$  in Eq. (11) to infinity, one obtains the neutrino additional energy in the local limit of weak interaction, the so-called Wolfenstein energy<sup>1</sup>. The additional energy obtained by this way is inapplicable in the case of charge-symmetric plasma, e.g. in the Early Universe. One has to take into account the additional contribution to the neutrino energy caused by the non-locality of weak interaction. This kind of energy was investigated in Refs. 6, 7, 8. The non-local correction to the Wolfenstein energy was taken in the form of the next terms in the expansion of the  $W$ - and  $Z$ -boson propagators by the inverse powers of their masses  $m_{W,Z}^{-2}$ . So, the first correction can be obtained from Eq. (11), if one retains the first term in the expansion of the functions  $F_{1,2}$  by  $m^{-2}$ . This correction has the form:

$$\Delta^{(1)} E^{\nu_\ell} = -\frac{16 G_F E}{3\sqrt{2}} \left( \frac{\langle E_{\nu_\ell} \rangle N_{\nu_\ell} + \langle E_{\bar{\nu}_\ell} \rangle \bar{N}_{\nu_\ell}}{m_Z^2} + \delta_{\ell e} \frac{\langle E_e \rangle N_e + \langle E_{\bar{e}} \rangle \bar{N}_e}{m_W^2} \right), \quad (12)$$

which coincides with the result of Ref. 6. Here,  $\langle E_{\nu_\ell} \rangle, \langle E_{\bar{\nu}_\ell} \rangle, \langle E_e \rangle, \langle E_{\bar{e}} \rangle$  are the average energies of plasma neutrinos, antineutrinos, electrons and positrons respectively. However, the correction of the type of Eq. (12) can be insufficient in the case of ultra-high neutrino or antineutrino energies. That is why it is interesting to obtain the neutrino self-energy operator with using the dependence of the propagators of gauge bosons on the momentum transferred.

### 3. Kinematically Possible Regions For Neutrino Radiative Conversion in Plasma

In the analysis of a kinematical possibility for the neutrino radiative conversion (3) there can be essential three physical parameters, namely: the energy of the initial

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neutrino  $E$ , the neutrino additional energy in plasma  $\Delta E$  and the effective photon (plasmon) mass  $m_\gamma$ . The existence of the neutrino additional energy leads to the appearance of the effective squared mass  $m_L^2$  of the left-handed neutrinos:

$$m_L^2 = \mathcal{P}^2 = (E + \Delta E)^2 - \mathbf{p}^2, \quad (13)$$

where  $\mathcal{P}$  is the neutrino four-momentum in plasma in the plasma rest frame, while  $(E, \mathbf{p})$  should denote the neutrino 4-momentum in vacuum,  $E = \sqrt{\mathbf{p}^2 + m_\nu^2} \simeq |\mathbf{p}|$ . Hereafter we neglect the vacuum neutrino mass  $m_\nu$ , because in real astrophysical situations where  $\Delta E$  could play any role,  $m_\nu$  is less than  $\Delta E$  and much less than  $m_\gamma$ .

A condition for the kinematic opening of the process (3) has the form of the following inequality: <sup>3,4</sup>

$$m_L^2 \simeq 2 E \Delta E > m_\gamma^2. \quad (14)$$

Because of the dependence of the neutrino additional energy  $\Delta E$  on the neutrino energy  $E$ , see Eqs. (10), (11), the inequality (14) could be non-trivial. Let us consider it for different astrophysical situations.

### 3.1. Nonrelativistic Cold Plasma

Let us consider first the high-energy neutrino propagation through the “cold” plasma of the Sun or of red giants, where the temperature is  $T \sim (10^7 - 10^8) \text{ K} \sim (10^{-3} - 10^{-2}) m_e$ , and the electron density is  $N_e \sim 10^{26} \text{ cm}^{-3}$ . The effective plasmon mass in these conditions takes the form:  $m_\gamma = \sqrt{4\pi\alpha N_e/m_e}$ . In this situation we can assume electrons to be nonrelativistic,  $k^\mu \simeq (m_e, \mathbf{0})$ , so that  $(p - k)^2 \simeq -2m_e E$ . The stellar substance is transparent for the neutrino radiation, thus the contribution for the neutrino additional energy from thermal neutrinos can be neglected.

In these conditions, the electron gas can be considered as degenerate with a good accuracy. As a result, an integration in the functions  $F_{1,2}(\mu_e, m_W)$ , see Eq. (10), reduces to a computation of the electron density,  $N_e = Y_e N_B$ , where  $Y_e$  is the electron fraction, and  $N_B$  is the baryon density. The additional energy for a neutrino and antineutrino is

$$\Delta E^{\nu_\ell, \bar{\nu}_\ell} = \sqrt{2} G_F N_B \left( \pm \frac{\delta_{\ell e} Y_e}{1 \pm 2m_e E (m_W)^{-2}} \mp \frac{1}{2} (1 - Y_e) \right). \quad (15)$$

Insertion of the complex  $W$ -boson mass,  $m_W^*$  is essential for the electron antineutrino only, to avoid a pole of  $\Delta E$  at  $E = m_W^2/(2m_e)$ . The analysis of the threshold inequality (14) for the electron neutrino reduces, in view of (15), to the investigation of the positiveness of the square trinomial with respect to the energy  $E$ . Assuming that inside of the Sun  $Y_e \simeq 0.6$ , we conclude that the inequality (14) is not satisfied for any neutrino energies.

In the earlier papers 3, 4 where the local limit of the weak interaction was used, it was concluded that the neutrino radiative conversion in the considered conditions is possible for neutrino energies  $E$  greater than threshold energy  $E_0 \simeq 10^7 \text{ GeV}$ .

One can see that taking account of the non-locality of the weak interaction leads to the total closing of the effect for the electron neutrino in the nonrelativistic “cold” plasma.

Consider now the possibilities for a trueness of the inequality (14) in the same conditions for other neutrino flavors. Note that the question about any observational realization of this process remains open.

The analysis of the inequality (14) for the electron antineutrino, where a real part of  $\Delta E$  should be taken, shows that the radiative neutrino conversion is possible for antineutrino energies greater than the threshold energy value,  $E > E_0 \simeq 0.6 \times 10^7$  GeV.

An imaginary part of  $\Delta E^{\bar{\nu}_e}$  deserves a separate analysis. In general, the non-zero imaginary part of a self energy means an instability of a particle. In the considered case it means that the electron antineutrino is unstable with respect to the process  $\bar{\nu}_e + e^- \rightarrow W^-$  on plasma electrons. Using the formula for the width of the process:

$$w = -2 \operatorname{Im} \Delta E, \quad (16)$$

one obtains from Eq. (15):

$$w(\bar{\nu}_e + e^- \rightarrow W^-) = 2\sqrt{2} G_F N_e E_0 \frac{\Gamma_W E_0 / m_W}{(E - E_0)^2 + (\Gamma_W E_0 / m_W)^2}, \quad (17)$$

where  $E_0 = m_W^2 / (2m_e)$ . Evaluation of a mean free path with respect to this process,  $\lambda = 1/w$ , for  $N_e \sim 10^{26} \text{ cm}^{-3}$ ,  $E \sim 10^7$  GeV provides  $\lambda \sim 100$  km, while in the maximum of the width defined by Eq. (17) at  $E = E_0$  one obtains  $\lambda \sim 200$  m. It is obvious, that the process  $\bar{\nu}_e + e^- \rightarrow W^-$  dominates the radiative neutrino conversion, see Refs. 3, 4. If one formally takes the limit  $\Gamma_W \rightarrow 0$  in Eq. (17) to obtain:

$$w(\bar{\nu}_e + e^- \rightarrow W^-) = 2\sqrt{2} \pi G_F N_e E_0 \delta(E - E_0). \quad (18)$$

It coincides with the result of a direct calculation of the  $W$ -boson production by  $\bar{\nu}_e$  scattered off nonrelativistic electron gas, without taking account of the instability of the  $W$ -boson.

The interaction of the  $\mu$ - and  $\tau$ -neutrinos with medium occurs only through the  $Z$ -boson exchange with the zero momentum transfer and, as it was pointed above, it is completely described by the local limit of the weak interaction. As it can be seen from Eq. (15), the  $\nu_\mu$ ,  $\nu_\tau$  additional energy is negative, consequently the neutrino radiative conversion process is closed for these neutrino flavors.

In turn, the antineutrino  $\bar{\nu}_\mu$  and  $\bar{\nu}_\tau$  additional energy is positive. To estimate the border of the kinematically possible region for the  $SL\nu$  process in this case one can use a simple inequality:

$$E > E_0 = 4 \sin^2 \theta_W \frac{Y_e}{1 - Y_e} \frac{m_W^2}{m_e}. \quad (19)$$

For  $Y_e \simeq 0.6$ , the process is kinematically opened for  $\mu$ - and  $\tau$ -antineutrino energies greater than  $E_0 \simeq 2 \times 10^7$  GeV.

### 3.2. Neutron Stars

The substance of a neutron star is transparent for the neutrino radiation, as in the previous case. Electrons in extremely dense neutron stars are ultra-relativistic, therefore  $\mu_e \simeq p_F \simeq 120 (N_e/(0.05 N_0))^{1/3}$  MeV, where  $p_F$  is the electron Fermi momentum, and  $N_0 = 0.16 \text{ fm}^{-3}$  is the typical nuclear density<sup>13</sup>. Due to the modern estimations, the temperature inside neutron stars does not exceed a part of MeV, so the electron gas can be considered to be degenerate and an approximation of the zero temperature can be used. In this case the electron density is  $N_e = \mu_e^3/(3\pi^2)$  and the square effective plasmon mass is  $m_\gamma^2 = 2\alpha\mu_e^2/\pi$ .

The additional energy for an electron neutrino under such conditions takes the following form:

$$\Delta E^{\nu_e} = \sqrt{2}G_F \left( -\frac{1}{2} (1 - Y_e) N_B + \frac{1}{2\pi^2} A(E, \mu_e) \right), \quad (20)$$

$$A(E, \mu_e) = \frac{1}{16E^3} \left[ 4Em_W^2 \mu_e (m_W^2 + 2E\mu_e) - (m_W^6 + 4E\mu_e m_W^4) \ln \left( 1 + \frac{4E\mu_e}{m_W^2} \right) \right]. \quad (21)$$

The analysis of the threshold inequality (14) with taking account of Eqs. (20), (21) indicates that the  $SL\nu$  process for the electron neutrino is forbidden in the conditions of a neutron star.

The similar analysis can be held for the antineutrino. The additional energy in this case is

$$\Delta E^{\bar{\nu}_e} = \sqrt{2}G_F \left( \frac{1}{2} (1 - Y_e) N_B - \frac{1}{2\pi^2} \bar{A}(E, \mu_e) \right), \quad (22)$$

$$\bar{A}(E, \mu_e) = \int_0^{\mu_e} k^2 dk \int_{-1}^1 \frac{(1-x)dx}{1 - \frac{2E(1-x)k}{m_W^2} - i \frac{\Gamma_W}{m_W}}. \quad (23)$$

This integral can be easily calculated analytically but the final expression is too cumbersome. From the analysis of the kinematically possible region (14), where a real part of  $\Delta E$  should be taken, we can conclude that the radiative conversion process (3) is permitted for the electron antineutrino for energies greater than the threshold value  $E_0 \simeq 8 \times 10^4 \text{ GeV}$ , for  $Y_e \simeq 0.1$ ,  $N_B \simeq 10^{37} \text{ cm}^{-3}$ .

A comparison of these conclusions with the results of Refs. 3, 4 shows that taking account of the non-locality of the weak interaction does not lead to any qualitative changes of the conclusions on kinematical possibilities of the radiative conversion for the electron neutrino and antineutrino in the conditions of a neutron star.

Again, as in the considered case of “cold” plasma, an imaginary part of  $\Delta E^{\bar{\nu}_e}$  means an instability of the electron antineutrino with respect to the process  $\bar{\nu}_e + e^- \rightarrow W^-$  on plasma electrons. A width of the process can be obtained from



Eqs. (16), (22), (23), but in a general case the expression is rather cumbersome. It is essentially simplified for high neutrino energies,  $E \gg m_W \Gamma_W / \mu_e$ , taking the form:

$$w(\bar{\nu}_e + e^- \rightarrow W^-) = \frac{G_F m_W^4 \mu_e}{2\sqrt{2} \pi E^2} \left(1 - \frac{m_W^2}{4\mu_e E}\right) \theta\left(E - \frac{m_W^2}{4\mu_e}\right). \quad (24)$$

Evaluation of a mean free path with respect to this process for  $\mu_e \simeq 120 \text{ MeV}$ ,  $E \simeq 5 \times 10^4 \text{ GeV}$  provides  $\lambda \sim 10^{-5} \text{ cm}$ . Domination of the process  $\bar{\nu}_e + e^- \rightarrow W^-$  over the radiative neutrino conversion in the neutron star conditions is undoubted, see Refs. 3, 4.

For  $\mu^-$ ,  $\tau^-$ -neutrino and antineutrino, as well as in the case of “cold” plasma, it is correct to use the local limit of the weak interaction. Substituting the additional energy for  $\ell = \mu, \tau$

$$\Delta E^{\nu_\ell, \bar{\nu}_\ell} = \mp \frac{G_F}{\sqrt{2}} (1 - Y_e) N_B, \quad (25)$$

and the plasmon mass in the case of a cold degenerate plasma

$$m_\gamma = \left(\frac{2\alpha}{\pi}\right)^{1/2} (3\pi^2 Y_e N_B)^{1/3} \quad (26)$$

into the threshold inequality (14), we come to the conclusion that for  $\nu_\mu$ ,  $\nu_\tau$  the radiative conversion process (3) is forbidden. For  $\bar{\nu}_\mu$ ,  $\bar{\nu}_\tau$  the process is kinematically permitted for the energies greater than

$$E > E_0 = \frac{2 \sin^2 \theta_W}{1 - Y_e} \left(\frac{3 Y_e}{\pi}\right)^{2/3} \frac{m_W^2}{N_B^{1/3}}. \quad (27)$$

Using for estimation the values  $Y_e \simeq 0.1$ ,  $N_B \simeq 10^{37} \text{ cm}^{-3}$ , we obtain  $E_0 \simeq 2 \times 10^4 \text{ GeV}$ .

### 3.3. Hot Plasma of a Supernova Core

In this case one needs to use the general expression for the neutrino  $\nu_\ell$  and antineutrino  $\bar{\nu}_\ell$  additional energy (11) with taking account of the scattering on all plasma components. The additional energy can be written as:

$$\begin{aligned} \Delta E^{\nu_\ell, \bar{\nu}_\ell} = & \sqrt{2} G_F \left\{ \mp \frac{1}{2} (N_n - \bar{N}_n) \pm (N_{\nu_e} - \bar{N}_{\nu_e}) \right. \\ & \pm (N_{\nu_\mu} - \bar{N}_{\nu_\mu}) \pm (N_{\nu_\tau} - \bar{N}_{\nu_\tau}) \\ & + \frac{T^3}{2\pi^2} \left[ \delta_{\ell e} (B(\pm\mu_e, m_W, T) - B(\pm\mu_e, m_W, -T)) \right. \\ & \left. \left. + \frac{1}{2} (B(\pm\mu_{\nu_\ell}, m_Z, T) - B(\pm\mu_{\nu_\ell}, m_Z, -T)) \right] \right\}, \quad (28) \end{aligned}$$

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where we use the notation

$$B(\mu, m, T) = -\frac{m^2}{ET} \left[ \text{Li}_2 \left( e^{-\frac{\mu}{T}} \right) + a \int_0^\infty \frac{dy}{\exp(y - \mu/T) + 1} \ln \left| 1 + \frac{y}{a} \right| \right]. \quad (29)$$

Here,  $\text{Li}_2(z)$  is the Euler dilogarithm, and  $a$  is the dimensionless parameter,  $a = m^2/4ET$ .

In the limit  $m_W^2 \gg 4ET$ , that is  $a \gg 1$ , assuming that plasma is not degenerate ( $\mu \sim T$ ), the integral in Eq. (29) can be represented as the series expansion that can be calculated analytically:

$$\begin{aligned} \int_0^\infty \frac{dy}{e^{-\mu/T} e^y + 1} \ln \left( 1 + \frac{y}{a} \right) &= e^{-\mu/T} \int_0^\infty \frac{y dy}{e^{-\mu/T} e^y + 1} - \frac{1}{2} e^{-2\mu/T} \int_0^\infty \frac{y^2 dy}{e^{-\mu/T} e^y + 1} \\ &+ \frac{1}{3} e^{-3\mu/T} \int_0^\infty \frac{y^3 dy}{e^{-\mu/T} e^y + 1} - \dots \end{aligned} \quad (30)$$

Taking into account that the Fermi integrals are expressed in terms of polylogarithms:

$$\int_0^\infty \frac{y^n dy}{e^{-\mu/T} e^y + 1} = -n! \text{Li}_{n+1} \left( -e^{\mu/T} \right), \quad (31)$$

and using the recurrent connections between the polylogarithms  $\text{Li}_n(x)$  and  $\text{Li}_n(x^{-1})$ , one obtains the following expression:

$$\begin{aligned} \Delta E^{\nu_e} &= \sqrt{2} G_F \left[ C_V^e \frac{\mu}{3\pi^2} (\mu^2 + \pi^2 T^2) - \frac{2}{3\pi^2} \frac{E}{m_W^2} \left( \mu^4 + 2\pi^2 \mu^2 T^2 + \frac{7\pi^4}{15} T^4 \right) \right. \\ &+ \frac{8}{5\pi^2} \frac{E^2 \mu}{m_W^4} \left( \mu^4 + \frac{10\pi^2}{3} \mu^2 T^2 + \frac{7\pi^4}{3} T^4 \right) \\ &\left. - \frac{64}{15\pi^2} \frac{E^3}{m_W^6} \left( \mu^6 + 5\pi^2 \mu^4 T^2 + 7\mu^2 \pi^4 T^4 + \frac{31}{21} \pi^6 T^6 \right) + \dots \right]. \end{aligned} \quad (32)$$

It is worthwhile to note that the similar expression can be written for the electron antineutrino. To write it down one has to make a change  $E \rightarrow -E$  in Eq. (32). In Fig. 3, the additional electron neutrino energy  $\Delta E$  is illustrated as a function of the initial neutrino energy  $E$ . It is demonstrated that taking account of only few terms in the series by the initial energy leads to an overestimation or understatement of the additional energy.

For a numerical estimation of the borders of the kinematically possible region for the  $SL\nu$  process in a general case with using of Eq. (28), let us take  $\mu_e \simeq 160$  MeV,  $\mu_\nu \simeq \mu_e/4 \simeq 40$  MeV, see e.g. Refs. 14 and 15. The analysis displays that the process is forbidden for neutrinos of all flavors. For all types of antineutrinos the effect becomes possible for energies greater than  $2 \times 10^4$  GeV.

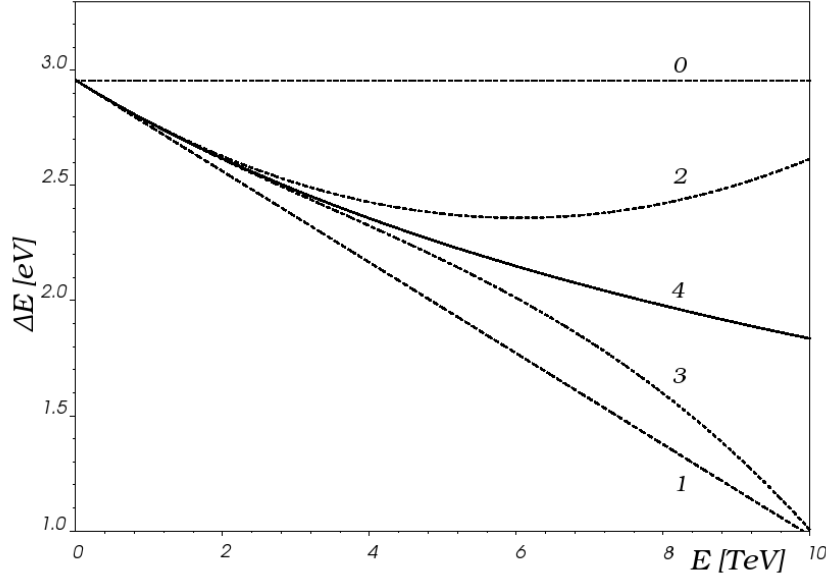


Fig. 3. Additional electron neutrino energy in the electron-positron medium ( $\mu_e \simeq 160$  MeV,  $T \simeq 30$  MeV) as an expansion into the series by initial neutrino energy: 0 is the local contribution; 1, 2 and 3 – with consecutive adding of non-local terms  $\sim E$ ,  $\sim E^2$  and  $\sim E^3$ ; 4 is the exact function.

As in the considered cases of “cold” plasma and of the neutron star interior, for electron neutrinos and antineutrinos the processes of the  $W$ -boson production on plasma electrons and positrons,  $\nu_e + e^+ \rightarrow W^+$  and  $\bar{\nu}_e + e^- \rightarrow W^-$ , are dominating. Using Eqs. (11), (16), one obtains the width of the process in the conditions of a hot dense plasma,  $\mu_e \sim T \gg m_e$ , for high neutrino energies,  $E \gg m_W \Gamma_W / \mu_e$ :

$$w(\bar{\nu}_e + e^- \rightarrow W^-) = \frac{G_F m_W^4 T}{2\sqrt{2} \pi E^2} \ln \left[ 1 + \exp \left( \frac{4\mu_e E - m_W^2}{4ET} \right) \right]. \quad (33)$$

Taking here the limit of cold plasma,  $T \rightarrow 0$ , one readily comes to Eq. (24). The width of the  $W^+$  production by  $\nu_e$  on positrons can be obtained from Eq. (33) by the replacement  $\mu_e \rightarrow -\mu_e$ .

Since in a dense plasma of the supernova core thermal neutrinos and antineutrinos of all flavors present, the processes of the  $Z$ -boson production should be also considered for the sake of completeness. Using Eqs. (11), (16), one obtains the width of the process where a high-energy antineutrino of the flavor  $\ell$  scatters off a thermal  $\nu_\ell$ :

$$w(\bar{\nu}_\ell + \nu_\ell \rightarrow Z) = \frac{G_F m_Z^4 T}{4\sqrt{2} \pi E^2} \ln \left[ 1 + \exp \left( \frac{4\mu_{\nu_\ell} E - m_Z^2}{4ET} \right) \right]. \quad (34)$$

The width of the process with a high-energy neutrino and a thermal antineutrino can be obtained from Eq. (34) by the replacement  $\mu_{\nu_\ell} \rightarrow -\mu_{\nu_\ell}$ . It should be noted that in the supernova core conditions  $\mu_{\nu_\ell} \simeq 0$  for  $\ell = \mu, \tau$ .

#### 4. Conclusion

We reexamine the previous results<sup>3,4</sup> on a possibility of the neutrino radiative conversion effect  $\nu_L \rightarrow \nu_R + \gamma$  (“spin light of neutrino”,  $SL\nu$ ) based on the additional neutrino energy in plasma, obtained in the local limit of the weak interaction (Wolfenstein energy) and with the first non-local correction. In the listed papers it was particularly demonstrated that the possibility of the  $SL\nu$  existence<sup>5</sup> is overstated and the process is kinematically forbidden in almost all real astrophysical conditions. The only question remained open whether this effect is possible in the case of ultra-high neutrino energies. In the present paper we eliminate this gap. Formulas for the neutrino and antineutrino additional energies in plasma are obtained, based on the  $W^-$  and  $Z$ -boson propagators depending on the momentum transferred. It should be noted that the question about any observational realization of the studied process requires a separate consideration. For high energy neutrinos and antineutrinos, the processes of the  $W^-$  and  $Z$ -boson production on plasma,  $\nu_e + e^+ \rightarrow W^+$ ,  $\bar{\nu}_e + e^- \rightarrow W^-$  and  $\bar{\nu}_\ell + \nu_\ell \rightarrow Z$ , are dominating.

#### Acknowledgments

This work was performed in the framework of realization of the Federal Target Program “Scientific and Pedagogic Personnel of the Innovation Russia” for 2009–2013 (State contract no. P2323) and was supported in part by the Ministry of Education and Science of the Russian Federation under the Program “Development of the Scientific Potential of the Higher Education” (project no. 2.1.1/13011), and by the Russian Foundation for Basic Research (project no. 11-02-00394-a).

#### References

1. L. Wolfenstein, *Phys. Rev. D* **17**, 9 (1978).
2. A. Studenikin, *J. Phys. A: Math. Gen.*, **39**, 6769 (2006).
3. A. V. Kuznetsov and N. V. Mikheev, *Mod. Phys. Lett. A*, **21**, 1769 (2006).
4. A. V. Kuznetsov and N. V. Mikheev, *Int. J. Mod. Phys. A*, **22**, 3211 (2007).
5. A. Studenikin, *J. Phys. A: Math. Gen.*, **41**, 164047 (2008).
6. D. Nötzold and G. Raffelt, *Nucl. Phys. B*, **307**, 924 (1988).
7. P. Langacker and J. Liu, *Phys. Rev. D*, **46**, 4140 (1992).
8. P. Elmfors, D. Grasso and G. Raffelt, *Nucl. Phys. B*, **479**, 3 (1996).
9. C. Lunardini and A. Yu Smirnov, *Nucl. Phys. B*, **583**, 260 (2000).
10. C. Lunardini and A. Yu Smirnov, *Phys. Rev. D*, **64**, 073006 (2001).
11. S. Sahu and W.-Y.P. Hwang, *Eur. Phys. J. C*, **58**, 609 (2008).
12. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
13. A. Yu. Potekhin, *Usp. Fiz. Nauk* **180**, 1279 (2010) [*Physics–Uspekhi* **53**, 1235 (2010)].
14. H.-Th. Janka, K. Langanke and A. Marek *et al.*, *Phys. Rept.*, **442**, 38 (2007).
15. F. S. Kitaura, H.-Th. Janka and W. Hillebrandt, *Astron. Astrophys.*, **450**, 345, (2006).